Recent results on QCD thermodynamics from Lattice

Sayantan Sharma

BROOKHAVEN
NATIONAL LABORATORY

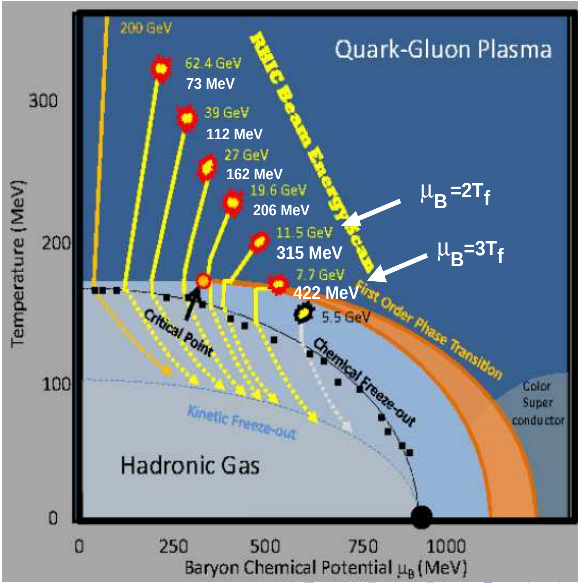
June 21, 2017

Outline

1. The QCD phase diagram: outstanding issues from lattice
2. Equation of state at finite μ_B
3. Critical-end point from Lattice
4. Lattice QCD Inputs for experiments

Outline

Major Themes from Lattice



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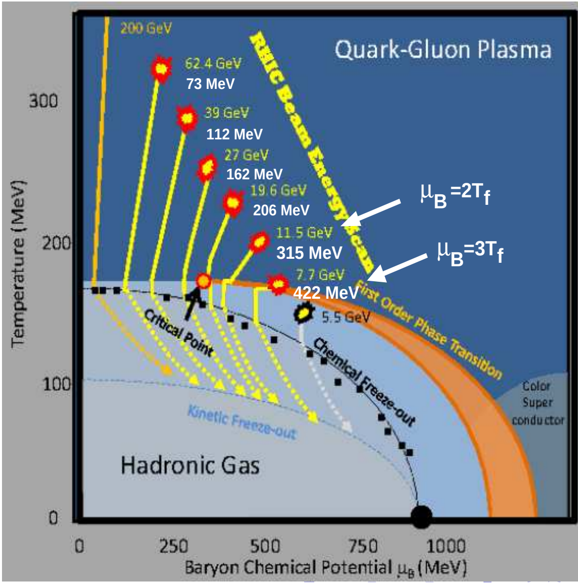
- In view of the RHIC Beam Energy Scan-II in 2019-20 it is important to have control over the Equation of State for $\mu_B/T \leq 3$.



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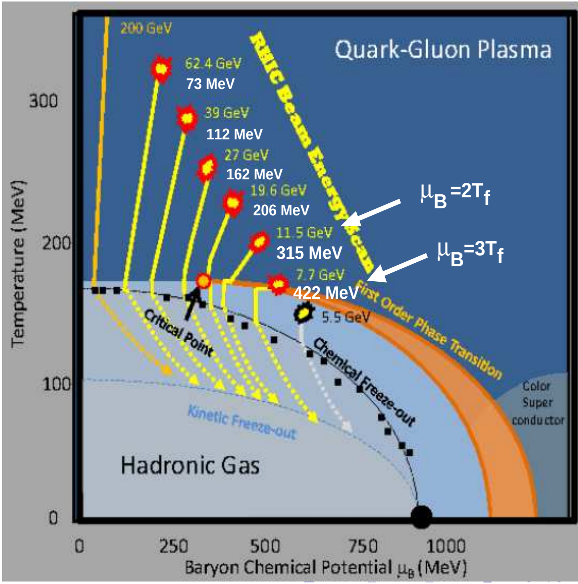

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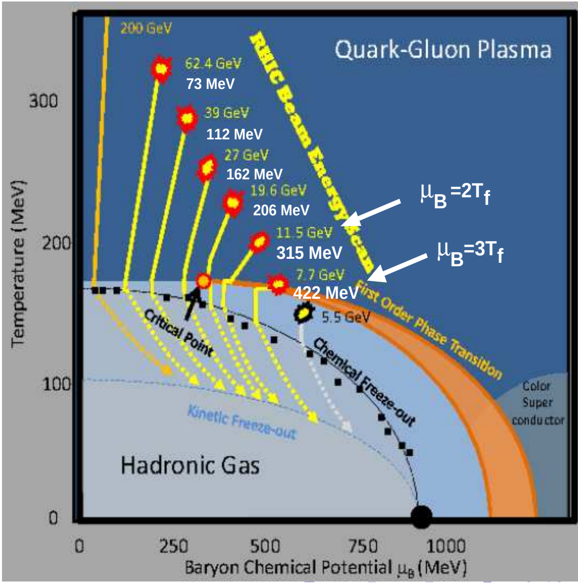
- Look for possible existence and bracket the position of critical end-point in the phase diagram.



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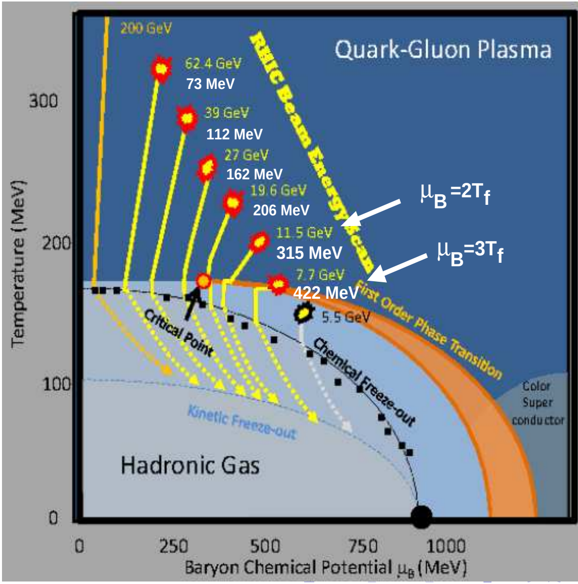
- Look for possible existence and bracket the position of critical end-point in the phase diagram.
- Provide inputs for heavy quark dynamics as a probe the QGP medium.

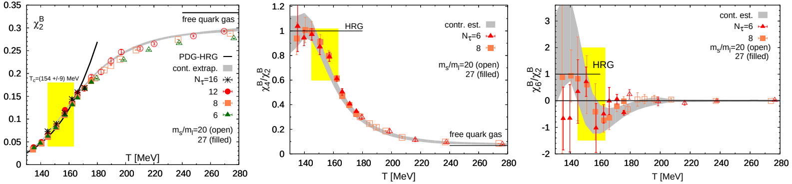


Outline

Basic methodology

- Traditional Monte-Carlo methods at finite μ_B suffer from sign problem.
- One of the most practical methods to circumvent it
  Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bi-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0,T)}{T^4} + \left(\frac{\mu_B}{T}\right)^2 \underbrace{\frac{\chi_2^B(0,T)}{2T^2}}_{P_2} + \left(\frac{\mu_B}{T}\right)^4 \underbrace{\frac{\chi_4^B(0)}{4!}}_{P_4} + \ldots$$

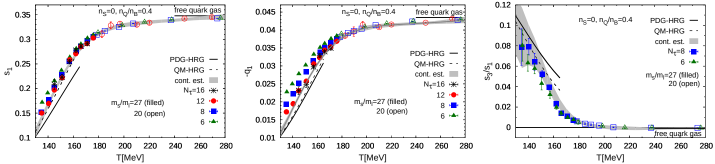


[Bi-BNL collaboration, 1701.04325]

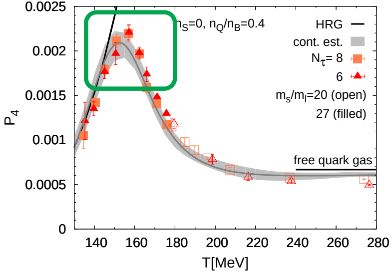
How to introduce constraints in EoS

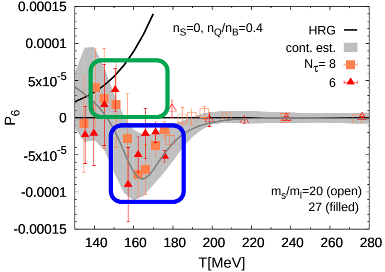
- In most central heavy-ion experiments typically:
  $n_S = 0$, Strangeness neutrality,
  $\frac{n_Q}{n_B} = \frac{n_P}{n_P + n_N} = 0.4$.
  [Bi-BNL collaboration, 1208.1220]
- For lower $\sqrt{s}$ collisions: Need to understand baryon stopping!
- Imposes non-trivial constraints on the variation of μ_S and μ_Q.
- Possible to vary them by only varying μ_B through

$$\begin{aligned} \mu_S &= s_1 \mu_B + s_3 \mu_B^3 + s_5 \mu_B^5 + \dots \\ \mu_Q &= q_1 \mu_B + q_3 \mu_B^3 + q_5 \mu_B^5 + \dots \end{aligned}$$



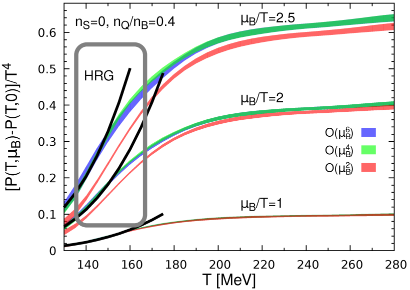
- Central values of P_4, P_6 already deviate from Hadron Resonance gas model at $T > 145$ MeV $\rightarrow$ need to reduce the errors on P_6 better.
- P_6 has characteristic structure at $T > T_c$ $\rightarrow$ remnant of the chiral symmetry due to the light quarks. Effects of $U_A(1)$ anomaly?
- Essentially non-perturbative $\rightarrow$ cannot be predicted within Hard Thermal Loop perturbation theory.

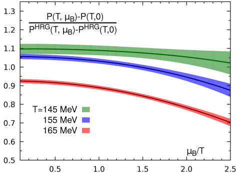




EoS in the constrained case

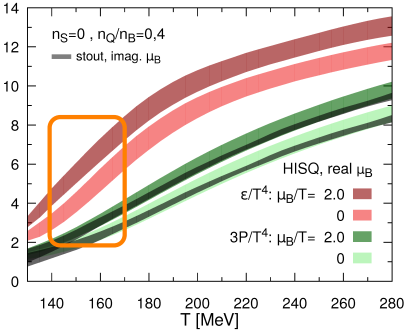
- The EoS for the constrained case is well under control for $\mu_B/T \sim 2.5$ with χ_6.
- Full parametric dependence for N_B on T available in arxiv: 1701.04325.
- Expanding to $\mu_B/T = 3$, need to calculate χ_8!

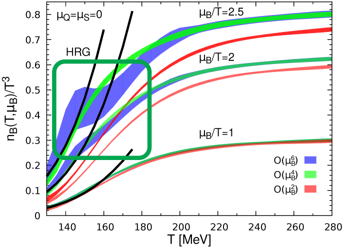




Summary for the EoS

- Continuum estimates from two different fermion discretizations and different methods of analysis agree for $\mu_B/T \leq 2$.
  [Bielefeld-BNL-CCNU collaboration, 1701.04325, Borsanyi et. al, 1606.07494].
- Steeper EoS for RHIC energies compared to LHC energy.

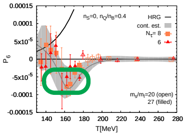


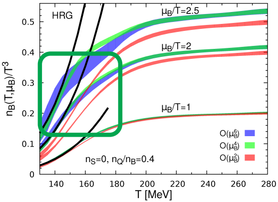


- χ_6 contribution is 30-times larger than in pressure.

$$\frac{N(\mu_B)}{T^3} = \frac{\mu_B}{T}\chi_2^B(0) + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^6 \chi_6^B(0) + \ldots$$

- Strongly sensitive to the singular part of χ_6^B.

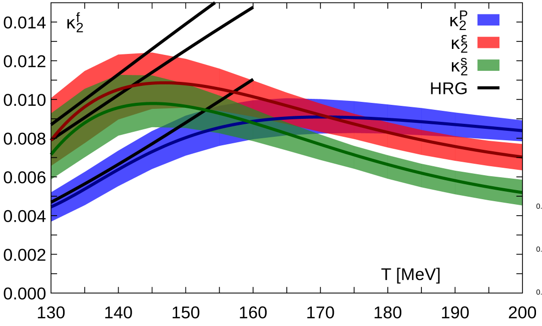


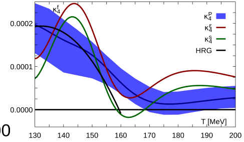


- For strangeness neutral system, effect is milder.

Curvature of freeze-out line

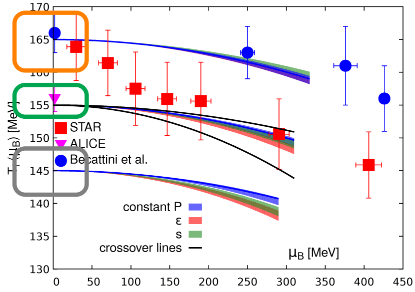
- The lines of constant $f \equiv \epsilon$ or p is characterized as:
  $T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T_0}\right)^4\right)$
- For $145 \leq T \leq 165$ MeV: $0.0064 \leq \kappa_2^P \leq 0.0101$, $0.0087 \leq \kappa_2^\epsilon \leq 0.012$.
- Consistent with the curvature of the chiral 'crossover' transition curve $0.0066(7)$ to $0.013(3)$. [arxiv:1011.3130, 1507.03571, 1507.07510, 1508.07599]
- For $\mu_B/T \leq 2$ the contribution from κ_4 to $T_f(\mu_B)$ within errors of κ_2.





Curvature of freeze-out line: Final summary

- Different LCP's agree within 2 MeV for $\mu_B/T \leq 2$ for 3 initial choices of T_0.
- For lines $P = \text{const}$, the entropy density changes by 15% $\rightarrow$ better description of LCP for viscous medium formed in heavy-ion collisions. [Bi-BNL-CCNU collaboration, 1701.04325].



- STAR results give a steeper curvature. arXiv:1412.0499.
- Agreement with the recent ALICE results. arXiv:1408.6403.
- Consistent with phenomenological models if a higher $T_f \sim 165$ is assumed Becattini et. al., 1605.09694.
  However lattice studies show explicitly that the HRG breaks down!

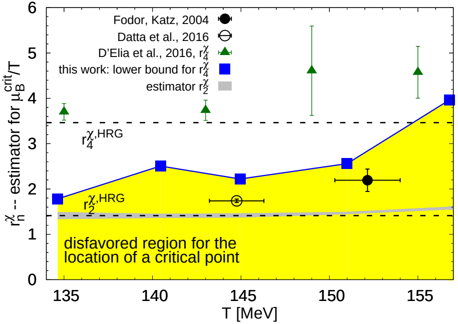
Outline

Critical-end point search from Lattice

- The Taylor series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume.
- The radius of convergence will give the location of the critical point. [Gavai& Gupta, 03]
- Definition: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
  - Strictly defined for $n \to \infty$. How large n could be on a finite lattice?
  - Signal to noise ratio deteriorates for higher order χ_n^B.

Critical-end point search from Lattice

- Different estimates from the ratios of fluctuations set a current bound for CEP to be $\mu_B/T > 2$ for $135 \leq T \leq 160$ MeV [Bielefeld-BNL-CCNU, 1701.04325].
- The χ_n^B extracted by analytic continuation using imaginary μ_B [D'Elia et. al., 1611.08285] are consistent with this bound.
- Some other lattice results gives a lower bound [Datta et. al., 1612.06673, Fodor and Katz, 04] $\longrightarrow$ need to understand the systematics in these studies. Ultimately all estimates will agree in the continuum limit!



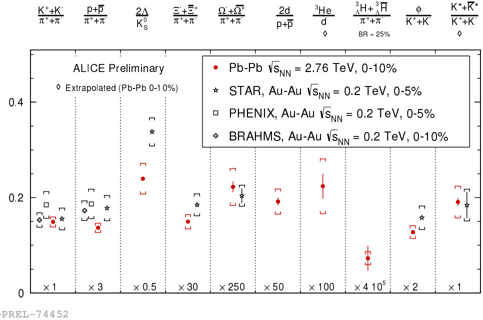
Outline

Characterizing Chemical Freezeout

- From the statistical fits to the hadron abundances:
  - $T_f = 156(2)$ MeV at $\sqrt{s} = 2.76$ TeV ALICE
- Fits to the particle abundances at ALICE included $\pi, K^{\pm}, K^0$ from excited charmed hadrons $\rightarrow$ could resolve p/π ratio discrepancy.

[A. Andronic et. al., 16]

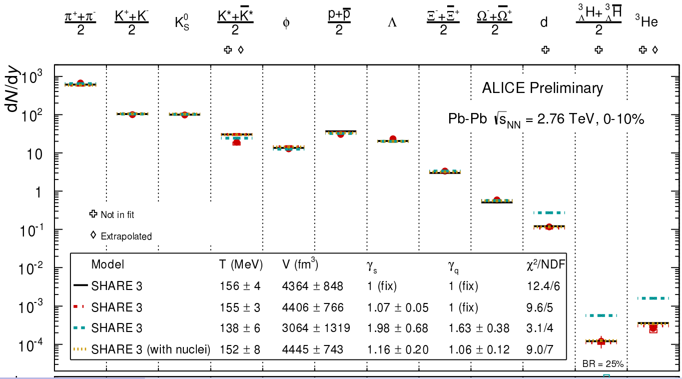
- Why are the estimates so much different?



Characterizing Chemical Freezeout

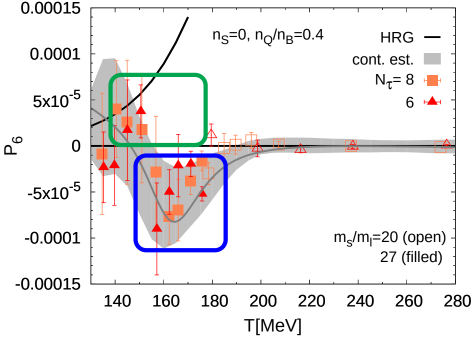
- Non-equilibrium effects for both light and strange baryons considered in detail through suppression factors γ.
- Gives even lower $T_f = 138(6)$ MeV.
- However such model overestimates light nuclei yields by a large factor!
  $\rightarrow$ particle yield in most central collisions consistent with thermal model fits!

[M Floris, QM 2014]



Freezeout and Hadron Resonance Gas model

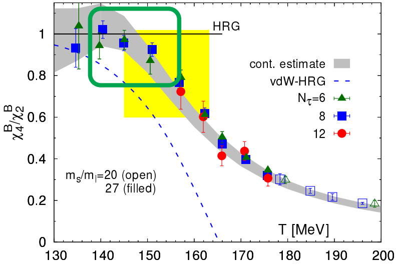
- T_f measured at ALICE is at the edge where lattice results deviate from HRG.
- For $T_f \sim 165$ MeV thermodynamic quantities deviate from HRG estimates

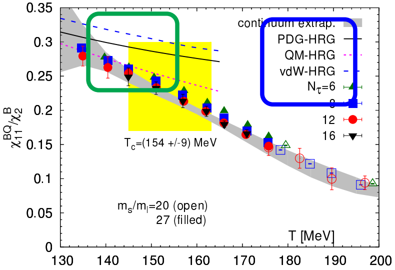


more dramatically!

- Repulsive baryon interactions more important? Excluded volume calculations included in the standard statistical model increases T_f for ALICE energies [A. Andronic et. al., 16] $\rightarrow$ Consistent with expected deviations from HRG model

Beyond HRG





[F. Karsch, QM17 proceedings]

- Including Van der Waal's interaction for baryons+non-interacting mesons+resonances, new versions of HRG has been studied $\rightarrow$ significant deviation from non-interacting HRG.

  [V. Vovchenko, M. I. Gorenstein and H. Stoecker 1609.03975]

- Lattice data can constrain such models strongly! Currently none of these models are perfect to describe QCD at freezeout.
- It would be important to resolve this 10 MeV spread in T_f specially for CEP searches.

Lattice Input to T_f

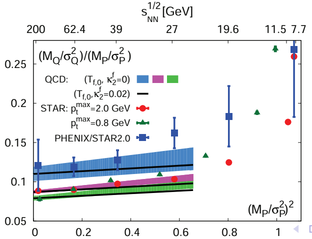
- Before directly comparing data from HIC experiments to lattice one has to take into account:
  - The expansion of the medium
  - the finite acceptance cuts in p_T
  - Unmeasured hadrons like neutrons.

- Choose observables in which such effects cancel each other
  $\Sigma_r^{QP} = \frac{R_{12}^Q}{R_{12}^P}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.
  [Karsch, Morita and Redlich, 15].

Lattice Input to T_f

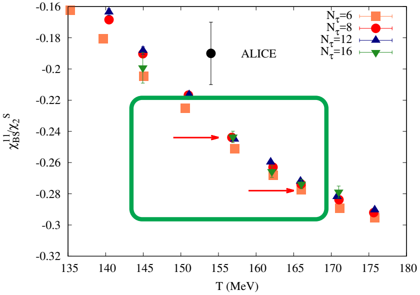
- For small μ_B/T, the freezeout curve:
  $T = T_{f,0}(1 - \kappa_2^f \mu_B^2/T_{f,0}^2)$.
- Major uncertainty : μ_B/T_f. Instead $\frac{n_B(\mu_B)}{\chi_2^B(\mu_B)} = \frac{\mu_B}{T} + \mathcal{O}(\frac{\mu_B^3}{T^3})$
- Performing a Taylor expansion:
  $\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0)\left[1 + c_{12}\ \left(R_{12}^B\right)^2\right] + \mathcal{O}\left(R_{12}^B\right)^4$
- Comparing with the lattice data for $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$ +assuming thermalization achieved under freezeout conditions:
  $T_f(\mu_B \sim 0) = 147(2)$ MeV for RHIC at $\sqrt{s} \sim 200$ MeV

  [Bielefeld-BNL-CCNU collaboration, 15]



New diagnostics!

- Off-diagonal fluctuations are more sensitive to deviation from HRG and baryon interactions.
- $\chi_{31}^{BS} - \chi_{11}^{BS}$ already rules out a different freezeout T_f for strangeness. [Bielefeld-BNL-CCNU collaboration, 13].



- χ_{11}^{BS}/χ_2^S shows $\sim$ 15% deviation between 155 and 165 MeV. Analysis with ALICE [A. Andronic et. al., 16] consistent with Lattice predictions at $\sim$ 155 MeV. Including $\Sigma^* \to N\bar{K}$ will make the ratio lower!
  Similar results from RHIC would be interesting! [A. Chatterjee et. al., Poster QM17]

From strangeness to charm at freezeout

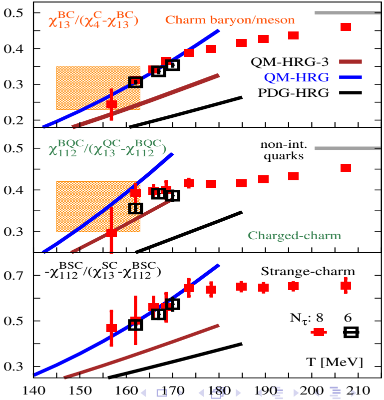
$$P(\mu_C, \mu_B, T) = P_M(T)\cosh\left(\frac{\mu_C}{T}\right) + P_{B,C=1}\cosh\left(\frac{\mu_B+\mu_C}{T}\right)$$

$$P_M = \chi_4^C - \chi_{13}^{BC}\ , P_{B,C=1} \sim \chi_{mn}^{BC}\ , m+n=4\ .$$

- Evidence of thermodynamic importance of yet to be measured charm baryons observed at T_f.

  [Bielefeld-BNL-CCNU collaboration, 14]

- To interpret experimental yields it is crucial to account for hadron abundances at T_f correctly.
- These resonances account for feed-down corrections.

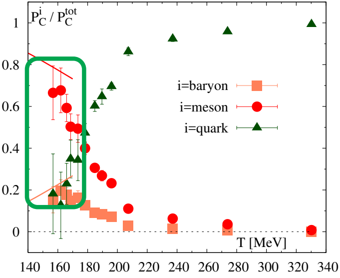


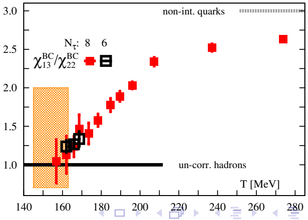
What are the charm degrees of freedom

- These techniques allow to single out charm baryon sector near $T_c \rightarrow$ studies conclude that open charm hadrons deconfine at T_c. Flavor hierarchy is disfavored. [Bielefeld-BNL collaboration, PLB, 14]
- However charm quarks remain correlated in the medium till about ~ 200 MeV $\rightarrow$ hints to presence of broad resonances.

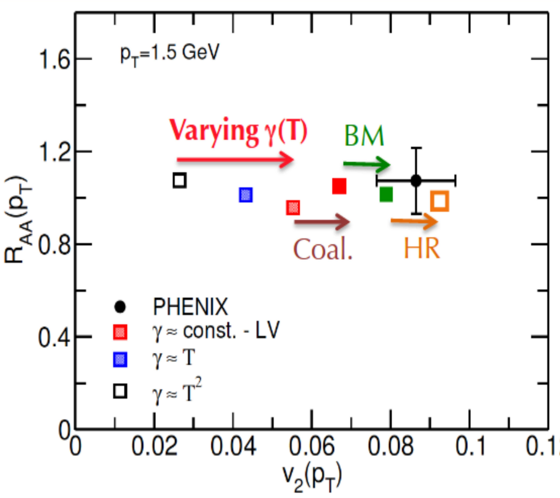
[Mukherjee, Petreczky, SS, PRD 2015, For phenomenology see M. He, R. J. Fries, R. Rapp, 12]

$$p_C = p_M \cosh\left(\frac{\mu_C}{T}\right) + p_{B,C=1} \cosh\left(\frac{\mu_C + \mu_B}{T}\right) + p_q(T) \cosh\left(\frac{\mu_C + \frac{\mu_B}{3}}{T}\right).$$



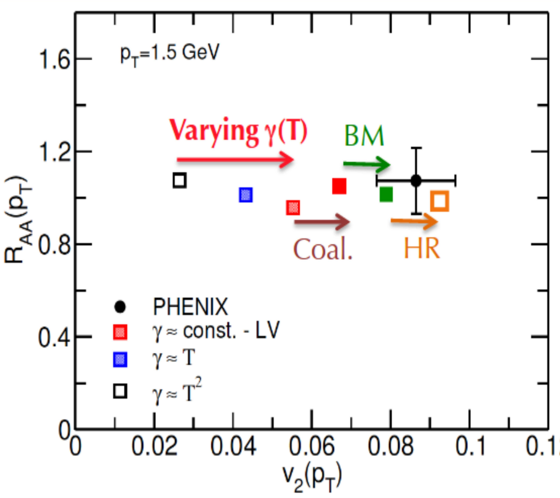


Implications for heavy flavor phenomenology



V. Greco's talk QM 17

- Lattice studies now predict that open charm hadrons melt at $T_c \Rightarrow$ freezeout temperature for D_s is now well known!



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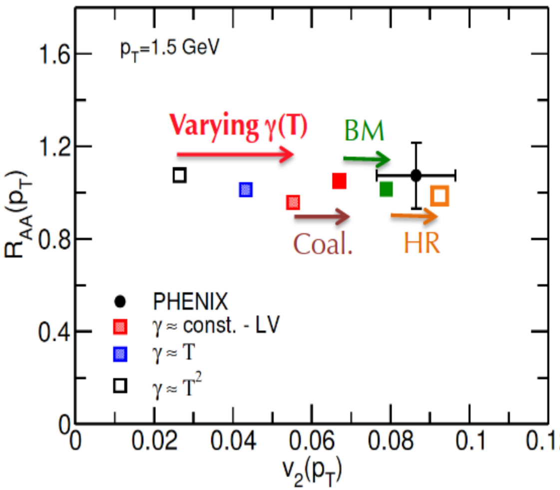

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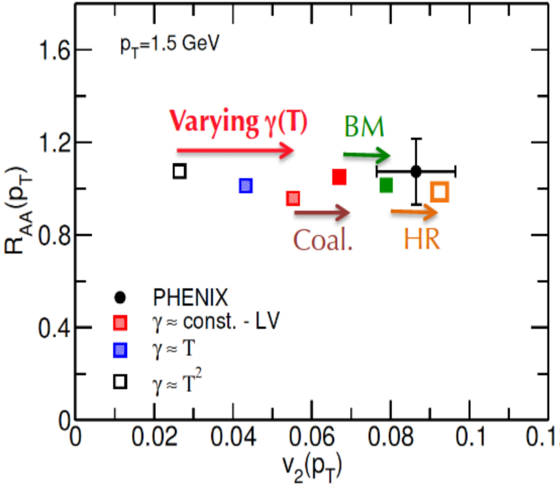
- Additional baryons may contribute to hadronic interactions near the freezeout $\longrightarrow$ can it explain the R_{AA} for open-charm mesons?
- Our study supports the picture of a broad D-meson resonance immediately beyond T_c as predicted from T-Matrix approach.

[M. He, R. J. Fries, R. Rapp, 12].



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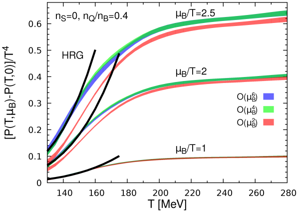
Conclusions and Outlook



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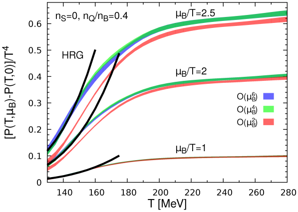

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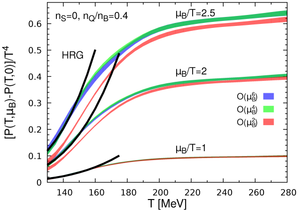

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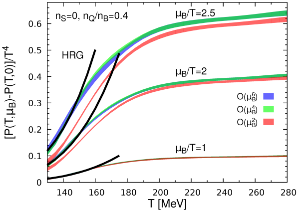

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- Higher order cumulants will also help in bracketing the possible CEP. Most LQCD calculations suggest $\mu_B(\text{CEP})/T \geq 2$.
- Beyond bulk thermodynamics, lattice results are now providing important insights for heavy-ion phenomenology.